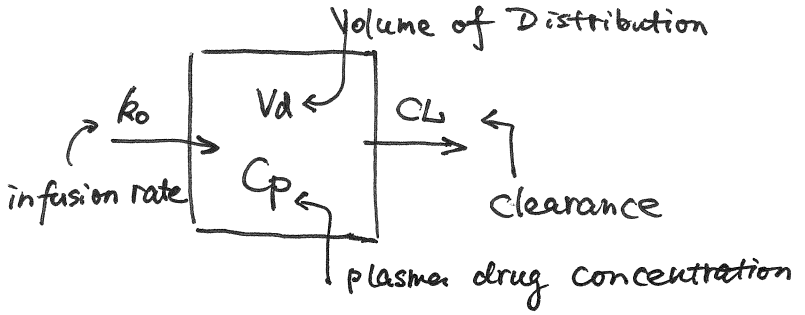
How to calculate C_p at time t for infusion?



$$V_d \frac{dC_p}{dt} = k_0 - CL \cdot C_p \xrightarrow{\text{Divide by "-CL"}} -\frac{V_d}{CL}\frac{dC_p}{dt} = -\frac{k_0}{CL} + C_p$$

"Amount of Drug" Change = "In" − "Out"

move around $\Longrightarrow$

$$\frac{dC_p}{-\frac{k_0}{CL} + C_p} = -\frac{CL}{V_d}\,dt \xrightarrow{\frac{CL}{V_d} = ke} \frac{dC_p}{-\frac{k_0}{CL} + C_p} = -ke\,dt$$

Integrate $\Longrightarrow$

$$\int_0^{C_p} \left(\frac{1}{-\frac{k_0}{CL} + C_p}\right) dC_p = \int_0^t -ke\,dt$$

Based on Integration Eq. $\Longrightarrow$

$$\ln\left|-\frac{k_0}{CL} + C_p\right|_0^{C_p} = -ke\,t\Big|_0^t$$

Because C_p always $< \frac{k_0}{CL}$ $\Longrightarrow$

$$\ln\left(-C_p + \frac{k_0}{CL}\right)\Big|_0^{C_p} = -ke\,t\Big|_0^t$$

$$\Longrightarrow \ln\left(-C_p + \frac{k_0}{CL}\right) - \ln\left(0 + \frac{k_0}{CL}\right) = -ke\,t$$

$$\Longrightarrow \ln\left(\frac{k_0}{CL} - C_p\right) - \ln\left(\frac{k_0}{CL}\right) = -ke\,t$$

$$\Rightarrow \ln\frac{\frac{k_0}{CL} - C_p}{\frac{k_0}{CL}} = -ke\,t \Rightarrow 1 - \frac{CL}{k_0}C_p = e^{-ke\,t}$$

$$\Longrightarrow C_p = \frac{k_0}{CL}\left(1 - e^{-ke\,t}\right) \Rightarrow C_p = \frac{k_0}{CL}\left(1 - e^{-\ln 2 \cdot \frac{t}{t_{1/2}}}\right)$$

($\frac{k_0}{CL}$: C_{ss})